SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces $ax^2+by^2+cz^2=1$ and $a^1x^2+b^1y^2+c^1z^2=1$.

17. (a) Find the coefficients of the direction which makes an angle $\frac{1}{2}\pi$ with the direction whose coefficients are (l,m,n).

(b) Show that curves $du^2-(u^2+a^2)dv^2=0$ on the right helicoids form on orthogonal set.

18. Derive the Liouville's formula for k_g.

19. Obtain the Second fundamental form of a surface.

20. State and prove Hilbert's lemma.

APRIL/MAY 2024

DMA33/GMA33 — DIFFERENTIAL GEOMETRY

Shanmuga Industries Arts & Science College T.V.Malai

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Torsion.

2. Write the Serret-Frenet formula.

3. Show that the family of curves $\phi(u,v)$ = constant are the solution of the differential equation $\phi_1 du+\phi_2 dv=0$.

4. Write the relation between E, F, G and H.

5. State Gauss-Bonnet theorem.

6. Prove that the Geodesic curvature vector of any curve is orthogonal to the curve.

7. State Rodrigues formula.

8. Define umbilics points.

9. Write the Gauss equations.

10. Write the Mainardi-Codazzi equations.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Find the arc length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $x=a\cosh\left(\frac{z}{a}\right)$ from the point $(a, 0, 0)$ to the point (x, y, z).

Or

(b) Find the curvature and torsion of the cubic curve given by $r=(u, u^2, u^3)$.

12. (a) Prove that, on the general surface, a necessary and sufficient condition that the curve $v=c$ be a geodesic is $EE_2+FE_1-2EF_1=0$.

Or

(b) Prove that if θ is the angle between the two direct at (u, v) on the two family of curves $Pdu^2+2\theta dudv+Rdv=0$ then $\tan\theta=\frac{2H(Q^2-PR)^{\frac{1}{2}}}{ER-2FQ+GP}$.

13. (a) Prove that if (λ,μ) is the geodesic curvature vector, then $k_g=\frac{-H\lambda}{Fu'-Gv'}=\frac{H\mu}{Eu'+Fv'}$.

Or

(b) Prove that if the orthogonal trajectories of the curves v = constant are geodesics, then $\frac{H^2}{E}$ is independent of u.

14. (a) Prove that the tangents to the edge of regression are characteristic kine.

Or

(b) Prove that the edge of regression of the rectifying developable has equation $R=r+k\frac{(\tau t+kb)}{k'\tau-k\tau'}$.

15. (a) Derive the Weingarten equations.

Or

(b) Derive compact surface whose points are umbilics.